Let's hold on to homeworks & quizzes & turn in at the end of class

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$$A = \int_M W \cdot N \, dA = \int_M W_1 \, dy \wedge dz + W_2 \, dz \wedge dx + W_3 \, dx \wedge dy$$

flux of W over M

$W = (\_\_\_ , \_\_\_ , \_\_\_)$ vector field

M surface = boundary of 3-d region

N = outward normal

ellipsoid

To do as a double integral: you would have to

ⓐ parametrize the ellipsoid $\phi(u,v) = (x, y, z)$ $x(u,v)$

ⓑ $W \cdot N \, dA = W(\phi(u,v)) \cdot (\phi_u \times \phi_v) \, du \, dv$

+ integrate

To do as a triple integral

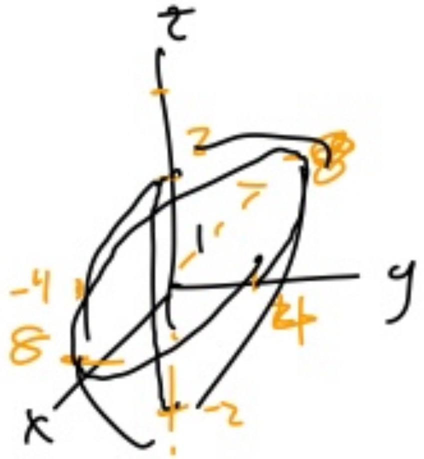
$$\int_M W \cdot N \, dA = \int_{\text{inside}} (\operatorname{div} W) \, d\text{vol} \leftarrow \text{triple integral}$$

$$\nabla \cdot W = \operatorname{div} W = \partial_x W_1 + \partial_y W_2 + \partial_z W_3$$

Example of integral over an ellipsoid.

$$x^2 + 4y^2 + 16z^2 \leq 64$$

standard form



$$\frac{x^2}{64} + \frac{\cancel{4}y^2}{\cancel{64}_{16}} + \frac{\cancel{16}z^2}{\cancel{64}_{4}} = 1$$

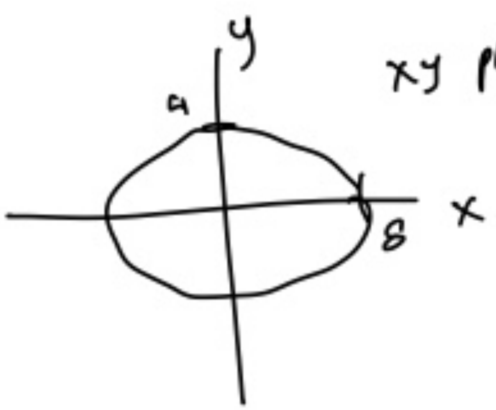
$$\frac{x^2}{8^2} + \frac{y^2}{4^2} + \frac{z^2}{2^2} = 1$$

$$\frac{z^2}{4} = 1 - \frac{x^2}{64} - \frac{y^2}{16}$$

$$z^2 = 4\left(1 - \frac{x^2}{64} - \frac{y^2}{16}\right)$$

$$z = \pm\sqrt{4\left(1 - \frac{x^2}{64} - \frac{y^2}{16}\right)}$$

$$\int \int \int_{z=-\sqrt{\ }}^{z=+\sqrt{\ }} (\operatorname{div} W)\, dz$$



xy plane

$$z=0 \quad \frac{x^2}{8^2} + \frac{y^2}{4^2} = 1$$

$$\frac{y^2}{4^2} = 1 - \frac{x^2}{64}$$

$$y^2 = 16\left(1 - \frac{x^2}{64}\right)$$

$$y = \pm\sqrt{16\left(1 - \frac{x^2}{64}\right)} =$$

$$\pm 4\sqrt{1 - x^2/64}$$

$$\int_{-8}^{8}\int_{y=-4\sqrt{1-\frac{x^2}{64}}}^{4\sqrt{1-\frac{x^2}{64}}}\int_{z=-\sqrt{4(1-\frac{x^2}{64}-\frac{y^2}{16})}}^{\sqrt{4(1-\frac{x^2}{64}-\frac{y^2}{16})}} (\operatorname{div} W)\,dz\,dy\,dx$$

$\int_\alpha V\cdot ds$ $\quad$ α circle $(x-1)^2+y^2=25$ ccw

V vector field $(x+y, x^2)$

- Need $\alpha(t)$
- $\int_\alpha V(\alpha(t))\cdot\alpha'(t)\,dt$

$x^2+y^2=1$
$x=\cos(t)$
$y=\sin(t)$

$x=5\cos(t)+1$
$y=5\sin(t)$

$$\alpha'(t)=(\underline{-5\sin(t)}, \underline{5\cos(t)})$$

$$V(\alpha(t))=(\underline{(5\cos(t)+1)+5\sin(t)}, \underline{(5\cos(t)+1)^2})$$

$$\int_\alpha V\cdot ds=\int_0^{2\pi}V(\alpha(t))\cdot\alpha'(t)\,dt=\int_0^{2\pi}\Big(-25\sin(t)\cos(t)-5\sin(t)-25\sin^2(t)+(5\cos(t)+1)^2\,5\cos(t)\Big)dt$$

Converting $\displaystyle\int_\alpha V \cdot ds$ $\qquad \alpha = \partial\Omega$ ← 2-d region

$$= \int_\Omega (\text{curl } V) \cdot N \, dA. \qquad \alpha = (x-1)^2 + y^2 = 25$$

N ↑ k

$$V = (x+y, x^2) \qquad \nabla \times V = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x+y & x^2 & 0 \end{vmatrix}$$

$$= 0i - 0j + k(2x-1)$$

$$= \int_{\text{inside}} (2x-1) \, dy \, dx$$

$$= \int_{x=-4}^{6} \int_{y=-\sqrt{25-(x-1)^2}}^{\sqrt{25-(x-1)^2}} (2x-1) \, dy \, dx$$

$(x-1)^2 + y^2 = 25$

$y^2 = 25 - (x-1)^2$

$y = \pm\sqrt{25-(x-1)^2}$